Study of $qqqc\bar{c}$ five quark system with three kinds of quark-quark hyperfine interaction

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The low-lying energy spectra of five quark systems $uudc\bar{c}$ (I=1/2, S=0) and $udsc\bar{c}$ (I=0, S=−1) are investigated with three kinds of schematic interactions: the chromomagnetic interaction, the flavor-spin dependent interaction and the instanton-induced interaction. In all the three models, the lowest five quark state ($uudc\bar{c}$ or $udsc\bar{c}$) has an orbital angular momentum $L = 0$ and the spin-parity $J^P = 1/2^-$; the mass of the lowest $udsc\bar{c}$ state is heavier than the lowest $uudc\bar{c}$ state.

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I. INTRODUCTION

The conventional picture of the proton and the corresponding excited states are a bound state of three light quarks uud in constituent quark model (CQM). Recently, an new measurement about parity-violating electron scattering (PVES) in JLab affords new information about the contributions of strange quarks to the charge and magnetization distributions of the proton, which provides a direct evidence of the presence of the multiquark components in the proton [1]. The importance of the sea quarks in the proton is also found in the measurement of the $\bar{d}/\bar{u}$ asymmetry in the nucleon [2].

Theoretically, the systematic investigation of baryon mass spectra and decay properties in CQM shows large deviations of theoretical values from the experimental data [3], such as the large $N\eta$ decay branch ratio of $N^*(1535)$ and the strong coupling of $\Lambda(1405)$ to the $\bar{K}N$. Riska and co-authors suggested that the mixtures of three-quark components qqq and the multiquark components $qqqq\bar{q}$ reduce these discrepancies [4–6]. In a recent unquenched quark model, by taking into account the effects of multiquark components via $^3P_0$ pair creation mechanism, it is also very encouraging to understand the proton spin problem and flavor asymmetry [7]. The $qqqq\bar{q}$ components could also be in the form of meson-baryon configurations, such as $N^*(1535)$ as a $K\Sigma$ bound state [8] and $\Lambda(1405)$ as a $\bar{K}N$ bound state [9].

In the early 1980s, Brodsky *et al*. proposed that there are non-negligible intrinsic $uudc\bar{c}$ components (∼ 1%) in the proton [10]. Later the study of Shuryak and Zhitnitsky show a significant charm component in $\eta^{'}$ also [11]. It is natural to expect the high excited baryons contain a large hidden charm five quark components too. Recently, some narrow hidden charm $N^*_{c\bar{c}}$ and $\Lambda^*_{c\bar{c}}$ resonances were predicted to be dynamically generated in the PB and VB channels with mass above 4 GeV and width smaller than 100 MeV [12, 13]. These resonances, if observed, definitely cannot be accommodated into the frame of conventional qqq quark models. A interesting question is whether these dynamically generated $N^*_{c\bar{c}}$ and $\Lambda^*_{c\bar{c}}$ resonances can be distinguished from penta-quark configuration states [4–6]. To distinguish the two hadron structure pictures, it should be worthwhile to explore the mass spectrum of the $qqqc\bar{c}$ consisted of the colored quark cluster $qqqc$ and $\bar{c}$.

The five quark configuration $qqqq\bar{s}$ and $qqqq\bar{c}$ with exotic quantum numbers have been extensively studied in the chiral quark model [14–17], colormagnetic interaction model [18, 19] and instanton-induced interaction model [20]. In this work, we study the mass spectra of the hidden charm systems $uudc\bar{c}$ and $udsc\bar{c}$ with three types of hyperfine interactions, color-magnetic interaction (CM) based on one-gluon exchange, chiral interaction (FS) based on meson exchange, and instanton-induced interaction ($Inst.$) based on the non-perturbative QCD vacuum structure.

This paper is organized as follows. In Section II, we show the wave functions of five quark states and Hamiltonians for the three types of interactions. In Section III, the mass spectra in the positive and negative sectors are presented. The paper ends with a brief summary.

II. THE WAVE FUNCTION AND HAMILTONIAN

As dealing with the conventional three quark model we need the wave functions and Hamiltonian to study the spectrum.

A. Wave functions of five quark systems

Before going to hidden-charm five quark $uudc\bar{c}$ and $udsc\bar{c}$ systems with isospin and strangeness as $(I, S) = (1/2, 0)$ and $(I, S) = (0, -1)$, respectively, we first consider the four quark subsystem, which can be coupled to an antiquark to form a hidden-charm five quark system. We use the eigenvalue method as given in Ref.[21] to derive the $udsc$ wave functions of the flavor symmetry $[211]_F$, $[22]_F$, $[4]_F$, $[31]_F$ and $[1111]_F$, which correspond to the $SU(4)$ flavor representation **15**, **20**, **35**, **45**, and **1**, respectively. For these flavor multiplets combined with $\bar{c}$, the following decomposition of the $SU(4)$ representation into $SU(3)$ representations can be found,

$$\begin{aligned} \mathbf{15} \times \bar{\mathbf{4}} &= \mathbf{8^0} + \mathbf{1^0} + \mathbf{8^0} + \mathbf{1^0} + \bar{\mathbf{3}}^{\mathbf{1}} + \mathbf{6^1} + \mathbf{15^1} \\ &+ \bar{\mathbf{3}}^{\mathbf{1}} + \bar{\mathbf{3}}^{\mathbf{2}} + \bar{\mathbf{6}}^{\mathbf{2}} + \bar{\mathbf{3}}^{\mathbf{1}} + \mathbf{3^{-1}} \end{aligned} \quad (1)$$

$$\begin{aligned}
\mathbf{20}\times\overline{\mathbf{4}} &= \mathbf{8^0}+\overline{\mathbf{10}}^{\mathbf{0}}+\overline{\mathbf{3}}^{\mathbf{1}}+\mathbf{6^1} \quad (2)\\
&+ \mathbf{15^1}+\mathbf{3^1}+\mathbf{15^1}+\overline{\mathbf{6}}^{\mathbf{-1}}+\mathbf{8^0}+\mathbf{6^0}\\
\mathbf{35}\times\overline{\mathbf{4}} &= \mathbf{10^0}+\mathbf{35^0}+\mathbf{24^1}+\mathbf{6^1}+\mathbf{15^2}+\mathbf{3^2} \quad (3)\\
&+ \overline{\mathbf{3}}^{\mathbf{4}}+\mathbf{15^{-1}}+\mathbf{10^0}+\mathbf{6^1}+\mathbf{3^2}+\mathbf{8^3}+\mathbf{1^3}+\mathbf{1^3}\\
\mathbf{45}\times\overline{\mathbf{4}} &= \mathbf{8^0}+\mathbf{10^0}+\mathbf{27^0}+\mathbf{24^1}+\mathbf{6^1}+\mathbf{15^1} \quad (4)\\
&+ \overline{\mathbf{3}}^{\mathbf{1}}+\mathbf{6^1}+\mathbf{15^2}+\mathbf{3^2}+\overline{\mathbf{6}}^{\mathbf{2}}+\mathbf{3^2}+\mathbf{8^3}+\mathbf{1^3}\\
&+ \mathbf{15^{-1}}+\mathbf{10^0}+\mathbf{8^0}+\mathbf{6^1}+\overline{\mathbf{3}}^{\mathbf{1}}+\mathbf{3^2},\\
\mathbf{1}\times\overline{\mathbf{4}} &= \mathbf{1^0}+\mathbf{3^1} \quad (5)
\end{aligned}$$

where the upper indexes denote the charm number. The decomposition notations in Ref. [22] are adopted. In the current work we only consider the hidden-charm five quark system, which means charm number C=0. The states lying in the octet $\mathbf{8^0}$ and singlet $\mathbf{1^0}$ of the $qqqc\bar{c}$ states carry the isospin and strangeness as (I,S)=$(1/2,0)$ and (I,S)=$(0,-1)$ for the octet, and (I,S)=$(0,-1)$ for the singlet, respectively, which are the states we need. The octet can be derived from $[211]_F$, $[22]_F$ and $[31]_F$. The singlet can be derived from $[211]_F$ and $[1111]_F$. Only can $[4]_F$ symmetry form deculplet when combined with antiquark $\bar{c}$, which does not contain the isospin and strangeness quantum numbers we want. The *uudc* wave function can be constructed directly by replacement rules mentioned in Ref. [23]. The explicit form of *uudc* and *udsc* wave functions are relegated to Appendix A. The phase convention is same as in Refs.[5, 6].

The general expression in the flavor-spin coupling scheme for these five quark wave functions is constructed as

$$\begin{aligned}
\psi^{(i)}(J,J_z) &= \sum_{a,b,c,d,e,f}\sum_{L_z,S_z,s_z} C^{[1^4]}_{[X^{(i)}]_f[CFS^{(i)}]_e} C^{[CFS^{(i)}]_e}_{[C^{(i)}]_d[FS^{(i)}]_c} C^{[FS^{(i)}]_c}_{[F]_a[S^{(i)}]_b}\\
&\cdot\ [X^{(i)}]_{f,L_z}[F^{(i)}]_{a,T_z}[S^{(i)}]_{b,S_z}\psi^C_{[211]_d}\\
&\cdot\ (S,S_z,L,L_z|\tilde{J},\tilde{J}_z)(\tilde{J},\tilde{J}_z,1/2,s_z|J,J_z)\\
&\cdot\ \bar{\xi}_{s_z}\varphi(r_{\bar{c}})\bar{\psi}^C\bar{\varphi}. \quad (6)
\end{aligned}$$

where $\tilde{J}$ is the total angular momentum of four quark and S the total spin of four quark, i is the number of the $qqqc\bar{c}$ configuration in both positive and negative parity sectors, which will be given explicitly later. $\bar{\psi}^C$, $\bar{\varphi}$ and $\bar{\xi}_{s_z}$ represent the color, flavor and spinor wave functions of the antiquark, respectively. $\varphi(r_{\bar{c}})$ represents the space wave function for antiquark. The symbols $C^{[.]}_{[..][...]}$ are S_4 Clebsch-Gordan coefficients for the indicated color-flavor-spin ($[CFS]$), color $\bar{\psi}^C$, flavor-spin ($[FS]$), flavor ($[F]$), spin ($[S]$), and orbital ($[X]$) wave functions of the *qqqc* system.

B. Hamiltonians

To investigate the mass spectrum of the five quark system, the non-relativistic harmonic oscillator Hamiltonian is introduced as in the light flavor case [24]:

$$\begin{aligned}
H &= \sum_{i=1}^{5}(m_i+\frac{\vec{p}_i^{\,2}}{2m_i})-\frac{\vec{P}_{cm}^2}{2M}\\
&+ \frac{1}{2}\sum_{i<j}^{5}(C[r_i-r_j]^2+V_0)+H_{hyp}. \quad (7)
\end{aligned}$$

where m_i denotes the constituent masses of quarks u,d,s,c (and the antiquark $\bar{c}$), and $\vec{P}_{cm}$ and M are the total momentum and total mass $\sum_{i=1}^{5} m_i$ of the five quark system. C and V_0 are constants. As pointed out by Glozman and Riska [23], one may treat the heavy-light quark mass difference by including a flavor dependent perturbation term $H_0^{''}$,

$$\begin{aligned}
H' &= \sum_{i=1}^{5}(m_i+\frac{\vec{p}_i^{\,2}}{2m})-\frac{\vec{P}_{cm}^2}{10m}\\
&+ \frac{1}{2}\sum_{i<j}^{5}(C[r_i-r_j]^2+V_0)+H_0^{''}+H_{hyp}. \quad (8)
\end{aligned}$$

with m denoting the u,d,s quark mass. The Hamiltonian may be rewritten as a sum of 4 separated hamiltonians in Jacobi coordinates. The perturbation term $H_0^{''}$ has the following form

$$\begin{aligned}
H_0^{''} &= -\sum_{i=1}^{4}(1-\frac{m}{m_c})\{\frac{\vec{p}_i^{\,2}}{2m}-\frac{m_c\vec{P}_{cm}^2}{5m(3m+2m_c)}\}\delta_{ic}\\
&- (1-\frac{m}{m_{\bar{c}}})\{\frac{\vec{p}_5^{\,2}}{2m}\}\delta_{5\bar{c}}, \quad (9)
\end{aligned}$$

where the Kronecker symbol δ_{ic} means that the flavor-dependent term is nonzero when the i^{th} quark of four quarks is charm quark. If the center-of-mass term is dropped, the matrix element of perturbation term on the harmonic oscillator state in the negative parity sector ($L=0$) will be

$$\langle H_0^{''}\rangle_{[4]_X[1111]_{CFS}[211]_C[31]_{FS}} = -\frac{3}{4}\delta, \quad (10)$$

where $\delta=(1-m/m_c)\omega_5$ with the oscillator frequency $\omega_5=\sqrt{5C/m}$. For other states considered in this work the matrix elements can be also written as such simple form.

The term H_{hyp} reflects the hyperfine interaction between quarks in the hadrons. In this work we consider three types of the hyperfine interactions, *i.e.*, flavor-spin interaction (FS) based on meson exchange, color-magnetic interaction (CM) based on one-gluon exchange, and instanton-induced interaction (*inst.*) based on the non-perturbative QCD vacuum structure.

The flavor-spin dependent interaction reproduces well the light-quark baryon spectrum, especially the correct ordering of positive and negative parity states in all the considered spectrum [25]. The flavor dependent interaction has been extended to heavy baryons sector in Ref. [23]. Given that $SU(4)$ flavor symmetry is broken mainly through the quark mass differences, the hyperfine Hamiltonian can be written as the following form [23, 26]

$$H_{FS} = -C_\chi\sum_{i,j}^{4}\frac{m^2}{m_im_j}\sum_{F=1}^{14}\vec{\lambda}_i^F\cdot\vec{\lambda}_j^F\vec{\sigma}_i\cdot\vec{\sigma}_j, \quad (11)$$

where σ_i and λ_i^F are Pauli spin matrices and Gell-Mann $SU(4)_F$ flavor matrices, respectively, and C_χ a constant phenomenologically 20~30 MeV. In the chiral quark model [25],

only the hyperfine interactions between quarks are considered while the interactions between the quarks and the heavy antiquark $\bar{c}$ are neglected.

The chromomagnetic interaction, which have achieved considerable empirical success in describing the splitting in baryon spectra [27], are intensively used in the study of multiquark configurations [22, 28–30]. A commonly used hyperfine interaction is as the following [29],

$$H_{CM} = -\sum_{i,j} C_{i,j} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (12)$$

where σ_i is the Pauli spin matrice, λ_i^c is the Gell-Mann $SU(3)_C$ color matrices, and $C_{i,j}$ the colormagnetic interaction strength. The quark-antiquark strength factors are fixed by the hyperfine splittings of the mesons. For an antiquark the following replacement should be applied [31]: $\vec{\lambda}^c \to -\vec{\lambda}^{c^*}$.

The instanton induced interaction, introduced first by 't Hooft [32] for [ud]-quarks and then extended to three flavor case [33] and four flavor case [34], is also quite successful in generating the hyperfine structure of the baryon spectrum. The nonrelativistic limit of the unregularized quark-quark 't Hooft interaction has the form [34–37],

$$\begin{aligned} H_{Inst} &= -4\mathcal{P}^{\mathcal{D}}_{S=0} \otimes [\mathcal{W}_{nn}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nn) + \mathcal{W}_{ns}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(ns) \\ &+ \mathcal{W}_{nc}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nc) + \mathcal{W}_{sc}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(sc)] \otimes \mathcal{P}^{\mathcal{C}}_{\bar{\mathbf{3}}} \\ &- 2\mathcal{P}^{\mathcal{D}}_{S=1} \otimes [\mathcal{W}_{nn}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nn) + \mathcal{W}_{ns}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(ns) \\ &+ \mathcal{W}_{nc}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nc) + \mathcal{W}_{sc}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(sc)] \otimes \mathcal{P}^{\mathcal{C}}_{\mathbf{6}}, \quad (13) \end{aligned}$$

where $\mathcal{W}_{f_1 f_2}$ is the radial matrix element of the contact interaction between a quark pair with flavors f_1 and f_2, $\mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(f_1 f_2)$ the projector onto flavor-antisymmetric quark pairs; $\mathcal{P}^{\mathcal{C}}_{\bar{\mathbf{3}}}$ and $\mathcal{P}^{\mathcal{C}}_{\mathbf{6}}$ the projectors onto color antitriplet and color sextet pairs, respectively; $\mathcal{P}^{\mathcal{D}}_{S=0}$ and $\mathcal{P}^{\mathcal{D}}_{S=1}$ the projectors onto antisymmetric spin-singlet and symmetric spin-triplet states, respectively. For a three quark system, only two quarks qq in a spin singlet state with the flavor antisymmetry can interact through the instanton induced interaction. Here, we phenomenologically consider the instanton-induced interaction of the nc and sc quark pairs, although some authors [38, 39] assume that the heavy flavor decouples when the quark gets heavier than the Λ_{QCD}.

III. MASS SPECTRA OF $uudc\bar{c}$ AND $udsc\bar{c}$ SYSTEMS

In this section, we present the numerical results for the low-lying spectra of the five quark systems of $uudc\bar{c}$ and $udsc\bar{c}$ with the hyperfine interaction given by the color-magnetic interaction , the flavor-spin interaction, and the instanton-induced interaction, respectively. For the kinetic part and the confinement potential part of the Hamiltonian, we take the parameters of Refs. [23, 24], *i.e.*, $m_u = m_d = 340$ MeV, $m_s = 460$ MeV, $m_c = 1652$ MeV and $C = m_u\omega_5^2/5$ with $\omega_5 = 228$ MeV.

All other parameters for three different hyperfine interactions are listed in Table I. For the FS model, the C_χ parameter is taken from Ref. [24]. For the CM model, we take the $C_{i,j}$ parameters of Ref. [29], determined by a fit to the charmed ground states. For the $Inst.$ model, the parameters are determined by a fit to the splittings between the baryon ground states $N(938)$, $\Delta(1232)$, $\Lambda(1116)$, $\Sigma^0(1193)$, $\Omega(1672)$, $\Lambda_c(2286)$, $\Sigma_c(2455)$, $\Xi_c^0(2471)$, $\Xi_c^{'0}(2578)$ and $\Xi_c^{*0}(2645)$. The fit yields a ratio of about $\mathcal{W}_{ns}/\mathcal{W}_{nn} \simeq 2/3$, which is the same as in Ref. [34]. The parameter V_0 for each model is adjusted to reproduce the mass of $N^*(1535)$ as the lowest $J^P = 1/2^-$ N^* resonance of penta-quark nature.

TABLE I: The parameters (in the unit of MeV) for three kinds of hyperfine interactions.

CM [29]	C_{qq}	20	C_{qs}	14	C_{qc}	4	C_{sc}	5
	$C_{q\bar{c}}$	6.6	$C_{s\bar{c}}$	6.7	$C_{c\bar{c}}$	5.5	V_0	-208
FS [24]	C_χ	21	V_0	-269				
$Inst.$	$\mathcal{W}_{nn}$	315	$\mathcal{W}_{ns}$	200	$\mathcal{W}_{nc}$	70	$\mathcal{W}_{sc}$	52
	V_0	-213						

With all these Hamiltonian parameters fixed and the wave functions of five quark system outlined in the Sect.II, the matrix elements of Hamiltonian for various five-quark states can be calculated.

For the $uudc\bar{c}$ and $udsc\bar{c}$ systems, the lowest states are expected to have all five quark in the spatial ground state of $[4]_X$ configuration and hence negative parity. For the construction of color-flavor-spin wave-functions, the convenient coupling schemes for the FS and CM models are different, *i.e.*, $[1111]_{CFS}[211]_C[f]_{FS}[f]_F[f]_S$ and $[1111]_{CFS}[f]_F[f]_{CS}[211]_C[f]_S$, respectively. The flavor-spin configurations for the $uudc\bar{c}$ and $udsc\bar{c}$ systems of spacial ground state $[4]_X$ for the FS and CM models are listed in Table II, where the configurations $|1'>$ and $|3'>$ are only for the $udsc\bar{c}$ system. For the $udsc\bar{c}$ system, the $[211]'_F$ and $[211]_F$ correspond to the Weyl Tableaus $\begin{array}{|c|c|}\hline u & c \\ \hline d & \\ \cline{1-1} s & \\ \cline{1-1}\end{array}$ and $\begin{array}{|c|c|}\hline u & s \\ \hline d & \\ \cline{1-1} c & \\ \cline{1-1}\end{array}$, respectively.

TABLE II: The flavor-spin configurations for the $uudc\bar{c}$ and $udsc\bar{c}$ systems of spacial ground state $[4]_X$ for the FS and CM models.

	FS model	CM model
$\|1'>$	$[31]_{FS}[211]'_F[22]_S$	$[211]'_F[31]_{CS}[211]_C[22]_S$
$\|3'>$	$[31]_{FS}[211]'_F[31]_S$	$[211]'_F[31]_{CS}[211]_C[31]_S$
$\|1>$	$[31]_{FS}[211]_F[22]_S$	$[211]_F[31]_{CS}[211]_C[22]_S$
$\|2>$	$[31]_{FS}[31]_F[22]_S$	$[31]_F[211]_{CS}[211]_C[22]_S$
$\|3>$	$[31]_{FS}[211]_F[31]_S$	$[211]_F[31]_{CS}[211]_C[31]_S$
$\|4>$	$[31]_{FS}[22]_F[31]_S$	$[22]_F[22]_{CS}[211]_C[31]_S$
$\|5>$	$[31]_{FS}[31]_F[31]_S$	$[31]_F[211]_{CS}[211]_C[31]_S$
$\|6>$	$[31]_{FS}[31]_F[4]_S$	$[31]_F[211]_{CS}[211]_C[4]_S$

The corresponding seven $udsc\bar{c}$ wave functions with spin-parity $1/2^-$ are $|1', 1/2^-\rangle$, $|1, 1/2^-\rangle$, $|2, 1/2^-\rangle$, $|3', 1/2^-\rangle$, $|3, 1/2^-\rangle$, $|4, 1/2^-\rangle$, and $|5, 1/2^-\rangle$. The five wave functions with spin-parity $3/2^-$ are $|3', 3/2^-\rangle$, $|3, 3/2^-\rangle$, $|4, 3/2^-\rangle$, $|5, 3/2^-\rangle$ and $|6, 3/2^-\rangle$. The one wave function with spin-parity $5/2^-$ is $|6, 5/2^-\rangle$. They form three subspace of $J^P = 1/2^-, 3/2^-$ and $5/2^-$, respectively.

The energies for these different configurations have been calculated with three kinds of hyperfine interactions and are listed in Table III.

TABLE III: Energies (in unit of MeV) of the $uds c\bar{c}$ and $uudc\bar{c}$ system of the spacial ground state with three kinds of hyperfine interactions for different flavor-spin configurations.

	CM		FS		$Inst$		
conf.	$udsc\bar{c}$	$uudc\bar{c}$	$udsc\bar{c}$	$uudc\bar{c}$	$udsc\bar{c}$	$uudc\bar{c}$	J^P
$\|1'>$	4404	$--$	4169	$--$	4211	$--$	$\frac{1}{2}^-$
$\|3'>$	4325	$--$	4169	$--$	4222	$--$	$\frac{1}{2}^-$
	4432	$--$	4169	$--$	4222	$--$	$\frac{3}{2}^-$
$\|1>$	4480	4372	4156	4017	4287	4125	$\frac{1}{2}^-$
$\|3>$	4441	4333	4200	4059	4322	4167	$\frac{1}{2}^-$
	4538	4430	4200	4059	4322	4167	$\frac{3}{2}^-$
$\|2>$	4552	4436	4182	4052	4347	4195	$\frac{1}{2}^-$
$\|4>$	4471	4368	4229	4096	4360	4202	$\frac{1}{2}^-$
	4572	4468	4229	4096	4360	4202	$\frac{3}{2}^-$
$\|5>$	4617	4508	4258	4133	4386	4237	$\frac{1}{2}^-$
	4585	4477	4258	4133	4386	4237	$\frac{3}{2}^-$
$\|6>$	4629	4526	4362	4236	4461	4322	$\frac{3}{2}^-$
	4719	4616	4362	4236	4461	4322	$\frac{5}{2}^-$

For subspaces of $J^P = 1/2^-$ and $3/2^-$, some non-diagonal matrix elements of Hamiltonians are not zero and lead to the mixture of the configurations with the same spin-parity. After considering the configuration mixing, the eigenvalues of the Hamiltonians of the five quark $udsc\bar{c}$ and $uudc\bar{c}$ systems in the spatial ground state are listed in Table IV. The corresponding mixing coefficients of the states with spin-parity $1/2^-$ for three different models are listed in Tables V-VII. The spin symmetry $[4]_S$ is orthogonal to the spin symmetry $[31]_S$ and $[22]_S$. There is no mixing between the configuration $[31]_{FS}[31]_F[4]_S$ and other 7 configurations.

TABLE IV: Energies (in unit of MeV) the $udsc\bar{c}$ and $uudc\bar{c}$ systems in the spatial ground state under three kinds of hyperfine interactions (*i.e.*, with configuration mixing considered).

	CM		FS		$Inst.$	
J^P	$udsc\bar{c}$	$uudc\bar{c}$	$udsc\bar{c}$	$uudc\bar{c}$	$udsc\bar{c}$	$uudc\bar{c}$
$\frac{1}{2}^-$	4273	4267	4084	3933	4209	4114
$\frac{1}{2}^-$	4377	4363	4154	4013	4216	4131
$\frac{1}{2}^-$	4453	4377	4160	4119	4277	4204
$\frac{1}{2}^-$	4469	4471	4171	4136	4295	4207
$\frac{1}{2}^-$	4494	4541	4253	4156	4360	4272
$\frac{1}{2}^-$	4576		4263		4362	
$\frac{1}{2}^-$	4649		4278		4416	
$\frac{3}{2}^-$	4431	4389	4184	4013	4216	4131
$\frac{3}{2}^-$	4503	4445	4171	4119	4295	4204
$\frac{3}{2}^-$	4549	4476	4263	4136	4362	4272
$\frac{3}{2}^-$	4577	4526	4278	4236	4416	4322
$\frac{3}{2}^-$	4629		4362		4461	
$\frac{5}{2}^-$	4719	4616	4362	4236	4461	4322

TABLE V: The mixing coefficients of the states with spin-parity $1/2^-$ under the CM interaction including the $q\bar{q}$ interaction.

$udsc\bar{c}$	$\|1'>$	$\|1>$	$\|2>$	$\|3'>$	$\|3>$	$\|4>$	$\|5>$
4273	-0.54	0.06	-0.02	0.84	-0.05	-0.01	0.01
4377	-0.05	0.61	0.08	-0.12	-0.77	-0.15	-0.11
4453	0.83	-0.03	0.10	0.52	-0.15	-0.09	0.03
4469	-0.07	-0.17	-0.20	-0.05	-0.11	-0.95	-0.09
4494	-0.02	0.46	0.64	-0.02	0.40	-0.30	0.36
4576	0.14	0.61	-0.55	0.06	0.45	-0.03	-0.31
4649	0.03	0.08	-0.48	-0.02	-0.11	0.02	0.87
$uudc\bar{c}$	$\|1>$	$\|2>$	$\|3>$	$\|4>$	$\|5>$		
4267	0.61	0.11	-0.77	-0.03	-0.12		
4363	0.31	0.37	0.24	0.82	0.17		
4377	0.36	0.57	0.34	-0.56	0.34		
4471	0.63	-0.57	0.45	-0.05	-0.26		
4541	0.07	-0.44	-0.15	0.03	0.88		

TABLE VI: The mixing coefficients of the states with spin-parity $1/2^-$ under the FS interaction.

$udsc\bar{c}$	$\|1'>$	$\|1>$	$\|2>$	$\|3'>$	$\|3>$	$\|4>$	$\|5>$
4084	-0.03	-0.75	-0.66	0	0	0	0
4154	0	0	0	0.39	-0.70	-0.58	0.12
4160	0.95	-0.22	0.21	0	0	0	0
4171	0	0	0	0.92	0.35	0.18	-0.06
4253	0	0	0	-0.03	0.42	-0.35	0.84
4263	-0.29	-0.62	0.73	0	0	0	0
4278	0	0	0	0.07	-0.46	0.71	0.53
$uudc\bar{c}$	$\|1>$	$\|2>$	$\|3>$	$\|4>$	$\|5>$		
3933	0.76	0.65	0	0	0		
4013	0	0	-0.78	-0.60	0.17		
4119	0	0	0.52	-0.47	0.71		
4136	0.64	-0.76	0	0	0		
4156	0	0	0.35	-0.65	-0.68		

For the lowest spatial excited states, one quark should be in p-wave, which results in a positive parity for the five quark system. For the $udsc\bar{c}$ system, there are thirty four wave functions with spin-parity $1/2^+$ and $3/2^+$, twenty two with $5/2^+$ and four with $7/2^+$. Similarly, there are too many states for $uudc\bar{c}$ system. Here, ten of all states with spin-parity $1/2^+$, five lowest states with spin-parity $3/2^+$, five lowest states with $5/2^+$, and all the states with spin-parity $7/2^+$ are listed Table VIII in terms of the energy.

While in the negative parity sector there are three subspaces for $1/2^-$, $3/2^-$ and $5/2^-$, respectively, for the positive parity sector, there are four subspaces for $1/2^+$, $3/2^+$, $5/2^+$ and $7/2^+$, respectively. In the process of the calculation, we take the L-S coupling scheme with standard Clebsch-Gordan coefficients of the angular momentum [40]. For the flavor-spin and instanton-induced interactions, due to the ignoring of quark-antiquark interaction, the $1/2^+$ and $3/2^+$ states of the same configuration $[f]_{FS}[f]_F[f]_S$ degenerate. In the CM model, the two states of the same configuration but different four

TABLE VII: The mixing coefficients of the states with spin-parity $1/2^-$ under the *Inst*. interaction.

$uds c\bar{c}$	$\|1'>$	$\|1>$	$\|2>$	$\|3'>$	$\|3>$	$\|4>$	$\|5>$
4209	0.99	-0.07	0.08	0	0	0	0
4216	0	0	0	0.97	0.12	0.02	0.19
4277	-0.04	-0.94	-0.35	0	0	0	0
4295	0	0	0	-0.19	0.86	-0.21	0.42
4360	-0.10	-0.34	0.93	0	0	0	0
4362	0	0	0	-0.07	0.13	0.97	0.19
4416	0	0	0	-0.10	-0.47	-0.12	0.87
$uud c\bar{c}$	$\|1>$	$\|2>$	$\|3>$	$\|4>$	$\|5>$		
4089	0.94	0.35	0	0	0		
4096	0	0	0.86	-0.20	0.47		
4157	0	0	0.11	0.97	0.20		
4175	-0.35	0.94	0	0	0		
4242	0	0	-0.50	-0.12	0.86		

TABLE VIII: Energies (in unit of MeV) of positive parity (L=1) $qqqc\bar{c}$ states with quantum numbers of N^*- and Λ^*-resonances under three kinds of interaction, with configuration mixing considered.

	CM		FS		*Inst*.	
J^P	$udsc\bar{c}$	$uudc\bar{c}$	$udsc\bar{c}$	$uudc\bar{c}$	$udsc\bar{c}$	$uudc\bar{c}$
$\frac{1}{2}^+$	4622	4456	4291	4138	4487	4396
$\frac{1}{2}^+$	4636	4480	4297	4140	4501	4426
$\frac{1}{2}^+$	4645	4557	4363	4238	4520	4426
$\frac{1}{2}^+$	4658	4581	4439	4320	4540	4470
$\frac{1}{2}^+$	4690	4593	4439	4367	4557	4482
$\frac{1}{2}^+$	4696	4632	4467	4377	4587	4490
$\frac{1}{2}^+$	4714	4654	4469	4404	4590	4517
$\frac{1}{2}^+$	4728	4676	4486	4489	4614	4518
$\frac{1}{2}^+$	4737	4714	4492	4508	4616	4549
$\frac{1}{2}^+$	4766	4720	4510	4515	4626	4566
$\frac{3}{2}^+$	4623	4457	4291	4138	4487	4396
$\frac{3}{2}^+$	4638	4515	4297	4140	4501	4426
$\frac{3}{2}^+$	4680	4561	4363	4238	4520	4426
$\frac{3}{2}^+$	4692	4582	4439	4320	4540	4470
$\frac{3}{2}^+$	4695	4625	4439	4367	4557	4482
$\frac{5}{2}^+$	4705	4539	4297	4140	4501	4426
$\frac{5}{2}^+$	4719	4649	4439	4320	4540	4470
$\frac{5}{2}^+$	4773	4689	4467	4367	4587	4482
$\frac{5}{2}^+$	4793	4696	4486	4404	4615	4490
$\frac{5}{2}^+$	4821	4710	4492	4515	4632	4517
$\frac{7}{2}^+$	4945	4841	4638	4508	4698	4566
$\frac{7}{2}^+$	4955	4862	4671	4551	4712	4634
$\frac{7}{2}^+$	4974	4919	4705	4587	4765	4669
$\frac{7}{2}^+$	5010		4759		4797	

quark angular momentum $\tilde{J}$ have a small splitting magnitude of several MeV as shown in Table VIII. Here only the masses of several lower energy states, which are more interesting to us, are listed in Table VIII.

The non-zero off-diagonal matrix elements introduce the mixture of the configurations with the same quantum number. The different hyperfine interactions give different admixture of configurations of certain state, which will result in different patterns of the electromagnetic and strong decays. The mixing effect has been explored in light quark sector, such as the decay of nucleon resonances $N^*(1440)$ [6] and $N^*(1535)$ [41].

For the $udsc\bar{c}$ system, in the CM model without $q\bar{q}$ interaction, the SU(3) flavor singlet with hidden charm, which has four quark configuration $[211]_F{}'[31]_{CS}[211]_C[22]_S$, is dominant in the lowest energy state, with a small admixture of $[211]_F[31]_{CS}[211]_C[22]_S$. The mixing of the two configurations is due to the flavor dependence of the $C_{i,j}$. After considering the $q\bar{q}$ interaction in CM model, the configuration $[211]_F{}'[31]_{CS}[211]_C[31]_S$ ($\sim$ 72%) becomes the dominant wave function component, with a strong admixture of $[211]_F{}'[31]_{CS}[211]_C[22]_S$ ($\sim$ 27%), as shown in Table V. The $q\bar{q}$ interaction leads to a further mixing of the two spin symmetry configurations of $[22]_S$ and $[31]_S$, besides the flavor symmetry breaking effects. In the FS model, the lowest state has a dominant four-quark configuration $[31]_{FS}[211]_F[22]_S$ ($\sim$ 42%), with a strong admixtures of $[31]_{FS}[31]_F[22]_S$ and $[31]_{FS}[211]'_F[22]_S$, as shown in Table VII. In the *Inst* model, the lowest state predominantly has the configuration $[211]_C[31]_{FS}[211]_F{}'[22]_S$, which is the same as the CM case without $q\bar{q}$ interaction.

For the $uudc\bar{c}$ system, there is no hidden charm SU(3) flavor singlet state. In the CM model after taking into account the $q\bar{q}$ interaction, the lowest energy state is mainly the admixture of $[211]_F[31]_{CS}[211]_C[31]_S$ ($\sim$ 67%) and $[211]_F[31]_{CS}[211]_C[22]_S$ ($\sim$ 27%), as shown in Table V. In the FS model, the lowest state is the four-quark configuration $[31]_{FS}[211]_F[22]_S$ ($\sim$ 52%), with a strong admixture of $[31]_{FS}[31]_F[22]_S$ ($\sim$ 42%). In the present *Inst*. model, assuming phenomenologically that the ′t Hooft's force also operates between a light and a charm quark, the configuration $[211]_F[31]_{CS}[211]_C[22]_S$ should be the lowest, as the spin $[22]_S$ and flavor $[211]_F$ contain more antisymmetrized quark pairs. In the *Inst* model, if it is assumed that the light quark and charm quark decouples, the $[211]_F[31]_{CS}[211]_C[22]_S$ and $[31]_F[31]_{CS}[211]_C[22]_S$ states degenerate and should be the lowest.

If the the flavor $SU(3)$ symmetry is restored and the light quark and charm quark decouples, the $udsc\bar{c}$ is lower than the $uudc\bar{c}$. For the positive parity $udsc\bar{c}$ states, under the CM interaction with the $q\bar{q}$ interaction, the lowest state has predominantly the four-quark configuration $[31]_F[31]_{CS}[211]_C[31]_S$, with a strong admixture of the configurations $[22]_F[31]_{CS}[211]_C[22]_S$ and $[31]_F[31]_{CS}[211]_C[22]_S$. In the FS model, the lowest positive parity state has predominantly the configuration $[4]_{FS}[22]_C[22]_S$. The *Inst* model predicts that the lowest state is the configuration $[1111]_F[31]_{CS}[211]_C[22]_S$, which can form the SU(3) flavor singlet state when combined with the antiquark.

Different hyperfine interactions predict different configurations for the lowest five quark states, which will result in different decay patterns and can be checked by future experiments.

IV. SUMMARY AND DISCUSSIONS

In this work we have estimated the low-lying energy levels of the five quark systems $uudc\bar{c}$ and $udsc\bar{c}$ with the hidden charm by using the three kinds of hyperfine interactions. The hidden charm states are obtained by diagonalizing the hyperfine interactions in each subspace with the same spin-parity. For the colormagnetic interaction, flavor-spin-dependent interaction and $Inst.$-induced interaction, all the models predict that the lowest states of the five quark systems $udsc\bar{c}$ and $uudc\bar{c}$ have the spin-parity $1/2^-$. The absolute value of the negative hyperfine energy for the configuration $[4]_{FS}[22]_F[22]_S$ in the positive parity sector is larger than the case of the $[31]_{FS}[211]_F{}'[22]_S$ in the negative parity sector. But this difference cannot overcome the orbital excited energy of the P-wave five quark system. This is in contrast with the situation in the light flavor sector with the chiral hyperfine interaction [24], due to the fact that the hyperfine splitting depends on the quark masses and gets weak for heavy quarks. In addition, for the flavor-spin interaction, the lowest $uudc\bar{c}$ state has negative parity, which is opposite to the lowest positive parity state of $uudd\bar{c}$ system containing only one heavy antiquark [15]. The four quarks $uudd$ with colored quark cluster configuration $[31]_X[4]_{FS}[22]_F[22]_S$ are strongly attractive due to the diquark structure $[ud][ud]\bar{c}$. However, the c quark in the diquarks $[ud][uc]$ with the same flavor-spin symmetry reduces to a large extent the hyperfine interaction energy. The instanton-induced interaction only operates on the color sextet and antitriplet diquark, and thus favors as well the similar diquark structure. The P-wave diquark-triquark structure $[ud][ud\bar{c}]$ is discussed under the colormagnetic interaction [19] and is almost as low as the $[ud][ud]\bar{c}$ [42]. It would be of interests to study the configurations of $[ud][uc\bar{c}]$ and $[ud][sc\bar{c}]$.

The coupled-channel unitary approach [12] predicted that the bound state $\bar{D}_s\Lambda_c$ is 30 ∼ 50 MeV lower than the bound state $\bar{D}\Sigma_c$. In the chiral quark model [13], there only exists the bound state $\bar{D}\Sigma_c$. In the present model, for the colored-cluster picture with three kinds of the residual interactions, the lowest $udsc\bar{c}$ system is heavier than the $uudc\bar{c}$ system. So the meson-baryon picture and the penta-quark picture give different prediction on the mass order of the super-heavy N^* and Λ^* with hidden charm.

In the CM model, the lowest $1/2^-$ and $3/2^-$ states, corresponding to the same four-quark configuration, are split by the quark-antiquark interaction. And the $3/2^-$ state of the $udsc\bar{c}$ and $uudc\bar{c}$ system is about 150 MeV heavier than the corresponding $1/2^-$ state. In the FS and $Inst.$ models, due to the lack of the quark-antiquark interaction, the two states degenerate.

In addition, we have also discussed the admixture pattern of the configurations with the same quantum numbers. The quark mass difference and quark-anti-quark interaction are the two sources of generating the configuration mixing, and the latter more important for the configuration mixing and mass splitting of penta-quark states. Since various configurations will result in different electromagnetic and the strong decays, the study of the decay properties may provide a good test of the models.

Experimental observation of the super-heavy N^* and Λ^* with hidden charm and their decay properties from $p\bar{p}$ reaction at PANDA and ep reaction at JLab 12 GeV upgrade are of great interests for our understanding dynamics of strong interaction.

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Appendix A: The wave functions for four quark subsystem

1. Flavor and spin couplings

Take the decomposition of the flavor-spin configuration $[31]_{FS}[211]_F[22]_S$ as an example,

$$|[31]_{FS1}\rangle = \frac{1}{\sqrt{2}}\{|[211]\rangle_{F1}|[22]\rangle_{S1} + |[211]\rangle_{F2}|[22]\rangle_{S2}\}\,, \tag{A1}$$

$$\begin{aligned}|[31]_{FS2}\rangle &= \frac{1}{2}\{-\sqrt{2}|[211]\rangle_{F3}|[22]\rangle_{S2} + |[211]\rangle_{F2}|[22]\rangle_{S2}\\ &- |[211]\rangle_{F1}|[22]\rangle_{S1}\}\end{aligned} \tag{A2}$$

$$\begin{aligned}|[31]_{FS3}\rangle &= \frac{1}{2}\{|[211]\rangle_{F1}|[22]\rangle_{S2} + |[211]\rangle_{F2}|[22]\rangle_{S1}\\ &+ |[211]\rangle_{F3}|[22]\rangle_{S1}\}\end{aligned} \tag{A3}$$

2. The flavor wave function of four quark subsystem *uudc*

The explicit forms of the flavor symmetry $[211]_F$

$$\begin{aligned}|[211]_{F1}\rangle &= \frac{1}{4}\{2uudc - 2uucd - duuc - uduc\\ &- cudu - ucdu + cuud + ducu\\ &+ ucud + udcu\}\,,\end{aligned} \tag{A4}$$

$$\begin{aligned}|[211]_{F2}\rangle &= \frac{1}{\sqrt{48}}\{3uduc - 3duuc + 3cuud\\ &- 3ucud + 2dcuu - 2cduu - cudu\\ &+ ucdu + ducu - udcu\}\,,\end{aligned} \tag{A5}$$

$$\begin{aligned}|[211]_{F3}\rangle &= \frac{1}{\sqrt{6}}\{cudu + udcu + dcuu\\ &- ucdu - ducu - cduu\}\,,\end{aligned} \tag{A6}$$

The explicit forms of the flavor symmetry $[22]_F$

$$\begin{aligned}|[22]_{F_1}\rangle &= \frac{1}{\sqrt{24}}[2uudc + 2uucd + 2dcuu + 2cduu\\ &- duuc - uduc - cudu - ucdu - cuud\\ &- ducu - ucud - udcu]\,,\end{aligned} \tag{A7}$$

$$\begin{aligned}|[22]_{F_2}\rangle &= \frac{1}{\sqrt{8}}[uduc + cudu + ducu + ucud\\ &- duuc - ucdu - cuud - udcu],\end{aligned} \tag{A8}$$

The explicit forms of the flavor symmetry $[31]_F$

$$\begin{aligned}|[31]_{F_1}\rangle &= \frac{1}{\sqrt{18}}[2uucd + 2cuud + 2ucud - cudu - ucdu\\ &- ducu - udcu - dcuu - cduu]\,,\end{aligned} \tag{A9}$$

$$\begin{aligned}|[31]_{F_2}\rangle &= \frac{1}{12}[6uudc - 3duuc - 3uduc - 4dcuu - 4cduu\\ &+ 5cudu + 5ucdu + 2uucd - cuud - ducu\\ &- ucud - udcu]\,,\end{aligned} \tag{A10}$$

$$\begin{aligned}|[31]_{F_3}\rangle &= \frac{1}{\sqrt{48}}[-3duuc + 3uduc - 3ducu + 3udcu\\ &- 2dcuu + 2cduu - cudu + ucdu - cuud\\ &+ ucud]\,.\end{aligned} \tag{A11}$$

3. The flavor wave function of four quark subsystem *udsc*

The explicit forms of the flavor symmetry $[31]_F$:

$$\begin{aligned}|[31]_{F_1}\rangle &= \frac{1}{\sqrt{12}}[ucsd - cdsu + uscd - dcsu + sucd\\ &- sdcu + cusd - dscu + scud - scdu\end{aligned}$$

$$+ \ csud - csdu] \,, \tag{A12}$$

$$\begin{aligned} |[31]_{F_2}\rangle &= \frac{1}{\sqrt{96}}\{3(usdc - sduc + ucds - cdus \\ &+ \ sudc - dsuc + cuds - dcus) \\ &+ \ 2(scdu - scud + csdu - csud) \\ &+ \ ucsd - cdsu + uscd - dcsu + sucd \\ &- \ sdcu + cusd - dscu\} \,, \end{aligned} \tag{A13}$$

$$\begin{aligned} |[31]_{F_3}\rangle &= \frac{1}{\sqrt{32}}\{2(udsc - dusc + udcs - ducs) \\ &+ \ sduc + usdc + cdsu + ucsd + cdus \\ &+ \ ucds + uscd + sdcu - dcsu - sucd - sudc \\ &- \ cuds - cusd - dsuc - dscu - dcus\} \end{aligned} \tag{A14}$$

The explicit forms of the flavor symmetry $[211]_F$:

$$\begin{aligned} |[211]_{F1}\rangle &= \frac{1}{4\sqrt{6}}\{3(sudc - sduc + usdc - dsuc \\ &+ \ sdcu - sucd + dscu - uscd) \\ &+ \ 2(csud - csdu + scud - scdu) \\ &+ \ cusd - cdsu + cdus - cuds \\ &+ \ dcus - ucds + ucsd - dcsu\} \,, \end{aligned} \tag{A15}$$

$$\begin{aligned} |[211]_{F2}\rangle &= \frac{1}{12\sqrt{2}}\{6(udsc - dusc) \\ &+ \ 5(dcsu - cdsu + cusd - ucsd) \\ &+ \ 4(scdu - csdu + csud - scud) \\ &+ \ 3(sduc - dsuc + usdc - sudc) \\ &+ \ 2(ducs - udcs) + cuds - ucds + sucd \\ &- \ sdcu + dscu - cdus + dcus - uscd\} \,, \end{aligned} \tag{A16}$$

$$\begin{aligned} |[211]_{F3}\rangle &= \frac{1}{6}\{2(udcs + cuds + dcus - ducs \\ &- \ cdus - ucds) + dcsu + uscd + scud \\ &+ \ sdcu + cusd + csdu - cdsu \\ &- \ ucsd - sucd - scdu - csud - dscu\} \end{aligned} \tag{A17}$$

The explicit forms of the flavor symmetry $[211]'_F$

$$\begin{aligned} |[211]'_{F1}\rangle &= \frac{1}{2\sqrt{3}}\{cdsu - ucsd + ucds - cdus \\ &+ \ cuds - scdu + dcsu - cusd \\ &+ \ dscu - csdu + csud - dcus\} \end{aligned} \tag{A18}$$

$$\begin{aligned} |[211]'_{F2}\rangle &= \frac{1}{6}\{2(udcs - sdcu + dscu - ducs + sucd - uscd) \\ &+ \ cdus - cdsu + ucds - ucsd + scud \\ &- \ scdu + dcsu - cuds + cusd \\ &- \ csud + csdu - dcus\} \end{aligned} \tag{A19}$$

$$\begin{aligned} |[211]'_{F3}\rangle &= \frac{1}{6\sqrt{2}}\{3(udsc + sudc + dsuc - dusc \\ &- \ sduc - usdc) + cdsu + ucsd + udcs + sucd \\ &+ \ scdu + cuds + csud + dscu + dcus - ducs \\ &- \ cdus - ucds - dcsu - uscd \\ &- \ scud - sdcu - cusd - csdu\} \end{aligned} \tag{A20}$$

4. The wave function of spin symmetry of four quark subsystem

The wave functions for spin symmetry $[22]_S$,

$$\begin{aligned} |[22]\rangle_{S1} &= \frac{1}{\sqrt{12}}\{2|\uparrow\uparrow\downarrow\downarrow\rangle + 2|\downarrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle \\ &- \ |\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\} \,, \end{aligned} \tag{A21}$$

$$|[22]\rangle_{S2} = \frac{1}{2}\{|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\} \,. \tag{A22}$$

More can be found in Ref. [43].